

## PRE-INFLATIONARY PERTURBATIONS SPECTRUM

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In the framework of a flat FLRW model we derive an inflationary regime in which the scalar field, laying on the plateau of its potential, admits a linear time dependence and remains close to a constant value.

The behaviour of inhomogeneous perturbations is determined on the background metric in agreement to the “slow-rolling” approximation. We show that the inhomogeneous scales which before inflation were not much greater than the physical horizon, conserve their spectrum (almost) unaltered after the de Sitter phase.

### 1. Introduction

The Standard Cosmological Model (SCM) finds many confirmations in the picture of the actual Universe <sup>1</sup>, but its shortcomings to describe very early stages of evolution appear as soon as the so-called horizon and flatness paradoxes are taken into account <sup>2,3,4</sup>.

The Inflationary Paradigm (IP) has acquired progressively an increasing interest, because it provides a natural explanation for such paradoxes <sup>2,3,4</sup>; the IP success relies overall on the consistent and simultaneous treatment of many different aspects of the cosmological puzzle. The capability to generate a Harrison–Zeldovich Spectrum (HZS) for the density perturbations outstands among these, and in fact the IP predicts it from the quantum fluctuations of the scalar field during the de Sitter phase.

This picture is well-grounded, but has to face the delicate point regarding the mechanism by which the quantum inhomogeneities approach a classical limit <sup>5</sup>. The quantum origin of the perturbations is also supported by the exponential suppression of the ultra-relativistic inhomogeneities during the de Sitter phase <sup>6</sup>.

In this work we show the existence of an inflationary regime allowing a classical origin for the HZS. In fact we deal with perturbations of the scalar field  $\phi$  which, if described by a HZS before inflation, survive to the Universe exponential expansion;

in our solution the inhomogeneities become super-horizon-sized and become seed for the structure formation when they re-enter the horizon after the IP.

## 2. Inflationary Regime

Let us consider a flat FLRW cosmology, summarized in a synchronous reference by the line element

$$ds^2 = c^2 dt^2 - a^2(t) \delta_{ij} dx^i dx^j, \quad i, j = 1, 2, 3 \quad (1)$$

where  $c$  denotes the speed of light.

Let us introduce the adimensional scalar field  $\xi = \sqrt{\chi} \phi$  (being  $\chi$  the Einstein constant) and analyse its dynamics on a plateau region described by the potential

$$V(\xi) = \frac{3\Lambda}{\chi c^2} - \frac{3\lambda}{\chi c^2} \frac{\xi^4}{4}; \quad (2)$$

this profile approximates a Coleman–Weimberg model and  $\Lambda$  is the scale of the symmetry breaking, while  $\lambda$  is a small coupling constant.

The dynamics of the coupled system  $(a, \xi)$  is summarized by the field equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\dot{\xi}}{6} + \Lambda - \frac{\lambda}{4} \xi^4 \quad (3a)$$

$$\ddot{\xi} + 3\frac{\dot{a}}{a}\dot{\xi} - 3\lambda\xi^3 = 0, \quad (3b)$$

where  $(\dot{\phantom{x}}) \equiv d/dt$ .

The solutions of (3) read as

$$a(t) = a_0 e^{H(t-t_0)}, \quad a_0, t_0 = \text{const.} \quad (4a)$$

$$\xi(t) = \alpha + \frac{\lambda}{H} \alpha^3 (t - t_0) \quad (4b)$$

with  $\alpha = \text{const.}$  ( $\alpha \ll \sqrt[4]{\Lambda/\lambda}$ ) and

$$H = \frac{1}{\sqrt{2}} \left[ \left( \Lambda - \frac{\lambda}{4} \alpha^4 \right) + \sqrt{\left( \Lambda - \frac{\lambda}{4} \alpha^4 \right)^2 + \frac{2}{3} \lambda^2 \alpha^6} \right]^{1/2} \sim \sqrt{\Lambda}. \quad (5)$$

The obtained de Sitter evolution holds as far as

$$t - t_0 \ll \frac{H}{\lambda \alpha^2} \sim \frac{\sqrt{\Lambda}}{\lambda \alpha^2} \quad (6)$$

and an e-folding of order  $\mathcal{O}(10^2)$  implies that

$$\alpha \ll \mathcal{O} \left( \frac{1}{10} \sqrt{\frac{\Lambda}{\lambda}} \right), \quad (7)$$

already ensured by the structure of the solution.

Here we got a consistent inflationary dynamics during which the scalar field remains

close to a constant value  $\phi \sim \alpha/\sqrt{\chi}$ .

When  $t - t_0$  increases enough, the scalar field enters a different regime of the IP, which ends with the fall-down into the true vacuum and the associated re-heating process.

### 3. Perturbations Dynamics

Now we study the behaviour of inhomogeneous perturbations of the scalar field, by neglecting their back-reaction on the scale factor  $a(t)$ .

If we take  $\xi \rightarrow \xi + \delta(t, x^i)$ , then this perturbation has to satisfy the dynamics

$$\ddot{\delta} + 3H\dot{\delta} - \left( \frac{c^2}{a^2} \delta^{ij} \partial_i \partial_j + 9\lambda\alpha^2 \right) \delta = 0; \quad (8)$$

the Fourier transform of (8) satisfies the equation

$$\ddot{\Delta} + 3H\dot{\Delta} + \left( \frac{|\vec{k}|^2}{a^2} c^2 - 9\lambda\alpha^2 \right) \Delta = 0 \quad (9)$$

where  $\Delta(t, \vec{k})$  is Fourier conjugated to  $\delta(t, x^i)$ . Equation (9) admits the (approximate) solution (with  $\dot{\Delta} \sim 0$ )

$$\Delta(t, \vec{k}) = \varepsilon(\vec{k}) \exp \left[ \frac{|\vec{k}|^2 c^2}{6H^2 a_0^2} \left( e^{-2H(t-t_0)} - 1 \right) + 3 \frac{\lambda}{H} \alpha^2 (t - t_0) \right] \quad (10)$$

where  $\varepsilon(\vec{k}) \equiv \Delta(t_0, \vec{k})$  denotes the pre-inflationary spectrum.

At the end of the de Sitter phase, we get the following (to leading order) spectrum  $\Delta_f(\vec{k})$  in terms of the initial one  $\Delta_i(\vec{k})$

$$\Delta_f(\vec{k}) \sim \Delta_i(\vec{k}) \exp \left( - \frac{k^2}{6H^2 a_0^2} \right); \quad (11)$$

all the physical scales  $\lambda_{ph} \equiv \frac{2\pi}{|\vec{k}|} a_0$ , which were not much greater than the physical horizon  $cH^{-1}$  at the beginning of the IP, survive with (almost) their spectrum.

If we deal with a HZS, the above analysis shows that its classical origin is compatible with an IP.

### References

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